

# COMBINED COMPETITIVE EXAMINATION (MAIN)

## MATHEMATICS-I

Time : 3 hours

Full Marks : 200

**Note :** (1) The figures in the right-hand margin indicate full marks for the questions.

(2) Attempt **five** questions in all.

(3) Question No. 1 is compulsory.

1. Answer any ten of the following :

4×10=40

(a) Find the eigenvalues and corresponding eigenvectors of the matrix

$$A = \begin{bmatrix} 3 & 1 \\ 2 & 4 \end{bmatrix}$$

(b) Prove that

$$\lim_{x \rightarrow \infty} \frac{\sin x}{x} = 0$$

(c) Show that

$$B(m, n) = B(m+1, n) + B(m, n+1)$$

(d) What are direction cosines of lines equally inclined to the axes? How many such lines are there?

(e) Find the singular solution of

$$y = x \frac{dy}{dx} + a \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$$

(f) Prove that the necessary and sufficient condition for the vector  $\vec{a}(t)$  to have constant magnitude is

$$\vec{a} \cdot \frac{d\vec{a}}{dt} = 0$$

