

ARUNACHAL PRADESH PUBLIC SERVICE COMMISSION

MATHEMATICS: PAPER - I

Time: 3 hours

Full marks 100

*(Group - A is compulsory. Attempt any FOUR questions from Group - B).*

GROUP - A

(This Group is COMPULSORY)

1: Attempt any ten (10) questions:

2 x 10=20

(i) The function  $f(x) = |x - 1|$  is

- (a) continuous at  $x = 1$
- (b) not derivable at  $x = 1$
- (c) both (a) and (b) are true
- (d) none of these

(ii) The value of the integral  $\int \frac{e^t}{1 + e^t} dt =$

- (a)  $1 + e^t$
- (b)  $\log(1 + e^t)$
- (c)  $[\log(1 + e^t)]^{-1}$
- (d)  $[1 + e^t]^{-1}$

(iii) The set  $\mathbb{N}$  of natural numbers is

- (a) bounded
- (b) bounded below
- (c) bounded above
- (d) not bounded

(iv) Every convergent sequence is

- (a) bounded
- (b) Cauchy
- (c) both (a) and (b) true
- (d) none of these

(v) A subgroup  $H$  of a group  $G$  is normal if and only if

- (a)  $xH \subset Hx^{-1}, \forall x \in G$
- (b)  $xH \subseteq Hx, \forall x \in G$
- (c)  $xHx^{-1} \supset H, \forall x \in G$
- (d)  $xHx^{-1} = H, \forall x \in G$

(vi) The set  $A = \{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \dots \dots\}$  is a

- (a) closed subset of  $R$ ,
- (b) open subset of  $R$ ,
- (c) both closed and open subset of  $R$ ,
- (d) neither closed nor open subset of  $R$ .

(vii) Let  $G = \{-1, 1\}$  be a group under multiplication. Define  $f : Z \rightarrow G$  by

$$f(n) = \begin{cases} 1, & \text{if } n \text{ is even;} \\ -1, & \text{if } n \text{ is odd} \end{cases}, \text{ where } (Z, +) \text{ is the group of integers. Then}$$

- (a)  $f$  is an isomorphism
- (b)  $f$  is an endomorphism
- (c)  $f$  is an automorphism
- (d)  $f$  is an epimorphism

(viii) The set  $S = \{a + ib, c + id\}$  is a basis of the space  $C(R)$  if and only if

- (a)  $ad - bc \neq 0$
- (b)  $ab - cd \neq 0$
- (c)  $ac - bd \neq 0$
- (d)  $ad - bc = 0$

(ix) Consider the mapping  $T : R^2 \rightarrow R^2$  such that  $T(1, 0) = (1, 1)$  and  $T(0, 1) = (-1, 2)$ . Then

- (a)  $T(x, y) = (x + y, x - 2y)$
- (b)  $T(x, y) = (x + y, x + 2y)$
- (c)  $T(x, y) = (x - y, x - 2y)$
- (d)  $T(x, y) = (x - y, x + 2y)$

(x) The matrix  $A = \begin{bmatrix} 1 & \alpha \\ \alpha & 1 \end{bmatrix}$  is invertible if

- (a)  $\alpha = \pm 1$
- (b)  $\alpha = \pm i$
- (c)  $\alpha \neq \pm 1$
- (d) none of these

(xi) The eigen values of the matrix  $A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$  are

- (a) 1, 6
- (b) 2, 3
- (c) 3, 4
- (d) 4, 6

(xii) The fuzzy set operations satisfy

- (a) Law of contradiction,
- (b) Law of excluded middle,
- (c) Both law of contradiction and law of excluded middle
- (d) None of the above property.

(xiii) The following linear programming problem:

Maximize  $z = x + y$  subject to the constraints:

$$x + y \leq 1, -3x + y \geq 3, x \geq 0, y \geq 0 \text{ has}$$

- (a) a unique optimal solution,
- (b) infinite number of optimal solution
- (c) no feasible solution
- (d) unbounded solution.

